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DESCRIPTION

OF THE

ANTIMETER:

OR, NEW

REFLECTING SECTOR,

And its Appendages, for meaſuring Angles
with Accuracy, by Sea or Land.

Together with

An Inveſtigation of its Theory.

To which is added,

NEW TABLES,

And Inſtructions for determining the

LONGITUDE

From the Moon's Altitude only.

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Of *Newington*, in *Surrey*, Inventor and Patentee.

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where the Inſtrument may alſo be had.

PREFACE.

THE Importance of a ſucceſsful Endeavour to produce an Inſtrument of ſuch extenſive Utility, as that *for meaſuring Angles accurately by Sea*, will be conſidered a ſufficient Stimulus to attempt it, and ample Reward when found by Experience to merit univerſal Attention.

The preſent Undertaking, therefore, ſeems to require neither Appeal or Comment. But the decided Opinion of that Part of Mankind, whoſe Wiſhes and Ideas aim at nothing beyond the blind Guides of Antiquity, is too frequently found a great Bar to Improvements, and was the only Difficulty the ingenious *Hadley* laboured under in the Introduction of his Inſtrument,

ſtrument, the Excellence of which being pointed out, removed that powerful Obſtacle, and effectually eſtabliſhed its Reputation.

In publiſhing any Thing new of this Kind, it therefore ſhould be the invariable Rule to ſubmit its Theory to the critical Enquiry of the Adept, who is, in effect, the Guardian of Science, and will not ſuffer an Impoſition on the Public to paſs unnoticed, nor deſcend to depreciate the Works of Merit.

The Matter only for the above Purpoſe, and ſuch as is neceſſary to accompany the Inſtrument, is to be found in the following few Pages, the Whole being reſtricted to what is pertinent, and improper Intruſions ſtudiouſly avoided: and further to obviate any Difficulty, which may appear againſt its immediate Reception, the Inventor has determined to apply himſelf for a Time ſolely to the Service of ſuch who wiſh to be inſtructed in the Uſe and Conſtruction of the Inſtrument.

The Table, N° I. for determining the Motion of the Object Glaſs, was calculated to its fulleſt Extent; but conceiving this Compendium moſt conſiſtent with the preſent Deſign; and being in ſome Meaſure induced thereto from the Uſe it will be found of in other Caſes, I have ſubſtituted the reducing Table, N° II. until it ſhall ſeem to meet Encouragement for an ample Publication. The Table of Refraction is carried to ſuch convenient Length, that the Defect in the Proportions mentioned by Dr. *Maſkelyne* is obviated, and the unneceſſary Part under three Degrees is omited.

The Tables VI. and VII. I preſume are entirely new, and from them the Moon's Parallax in Altitude may be found to a Second if neceſſary. The Method of finding the Longitude from the Moon's Altitude only, is rendered familiar, which by Help of the Inſtrument and Tables here preſented, may be practiſed at Sea as often as the Horizon is fit for Obſervations of that Nature, to as great a Certainty as that now uſed by three Co-temporary Obſervations. And it may be further obſeved, that the Moon's Altitude alone may frequently be taken when Co-temporary Obſervations cannot be made.

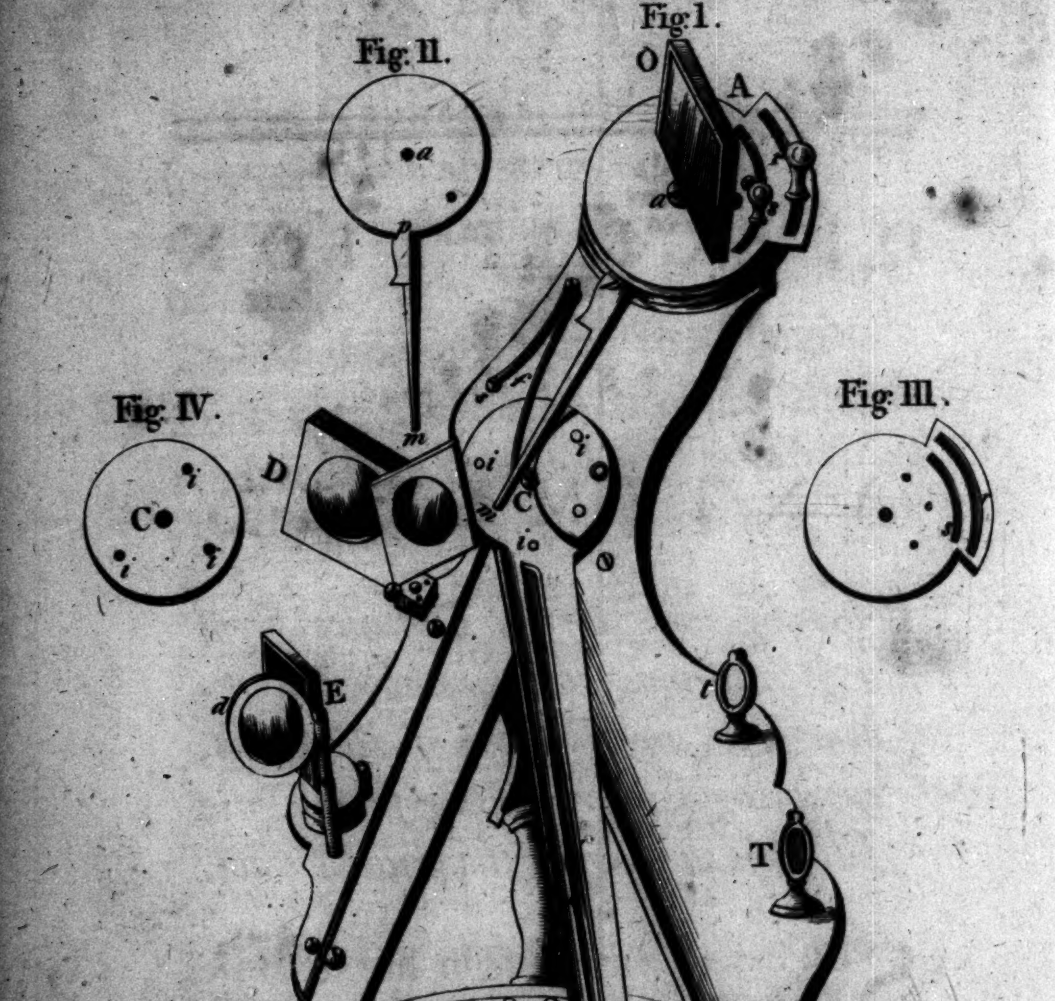
Fig. I.
Fig. II.
Fig. III.
Fig. IV.
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DESCRIPTION

OF THE

ANTIMETER,

Whereby the Moon's Diſtance from any Star may be obſerved on Shipboard, to any Degree of Accuracy, excepting Errors ariſing from Optic Illuſions in judging the true Contact.

SIR ISAAC NEWTON, in his OPTICS, Book I. Part I. Page 98, ſays, "If the "Theory of making Teleſcopes could at "Length be fully brought into Practice, "yet there would be certain Bounds beyond "which Teleſcopes could not perform; for "the Air through which we look upon the "Stars, is in a perpetual Tremor." So likewiſe it may be ſaid, that, if an Inſtru-

-ment for meaſuring Angles was ever ſo accurately formed, yet the true angular Diſtance between two Stars could never be abſolutely determined; for the Air is perpetually altering its Denſity, and it has frequently been obſerved, that at equal Altitudes, at the ſame Time it will be different, and thereby the general Laws of Refraction are diſturbed.

Illuſions of this Nature are ſuch as are alluded to in the preceeding Title, and therefore ought not to be conſidered as any Deficiency in the Perfection pretended to in the Inſtrument itſelf.

The moſt approved Inſtrument at preſent for the Purpoſe which this is intended, is, I believe, the Sextant on the Plan of *Hadley*'s Octant, which, with the Addition of the *Nonius*, or Micrometer Screw, together with Mr. *Martin*'s Adjuſtment of the Glaſſes is undoubtedly a noble and uſeful Inſtrument, ſufficient for all the common Purpoſes of Navigation.

But as in long Voyages, and in ſettling the Latitude and Longitude of Places, in the aſtronomical Part, the utmoſt Accuracy is required, it is to be feared that many Incorrections are hid in our Dependence on the Graduations of the Limb, frequently rendered inaccurate by the ſwelling, ſhrinking, or warping of the Inſtrument itſelf; and it is a Complaint too often heard of at Sea (where ſmall Inſtruments only are admitted) that the want of Space renders it inconvenient to read off to ſingle Minutes; and although ſome may be divided to half Minutes, yet they cannot be made uſe of without the Aſſiſtance of a good Magnifier; my endeavour has been to obviate Inconveniencies of this Nature, to give ſuch a Form to the Inſtrument, as will not partake of the caſual Errors above-mentioned, and to enable the Obſerver, with a weak Light, to read off as correct as he can make his Obſervation.

Various Attempts to effect the Purpoſe of multiplying the Space have been made, as by Wheel-work, and ſuch like mechanic Powers, but in general proved unſucceſsful, from the Deficiency in the mechanic Arts, in Relation to Theory; for although the whole Revolution of Wheels is uniform, yet the Infiniteſimals of their Motion are irregular, from the mechanical Conſtruction of their Teeth, wherefore, the finer the Teeth are, the nearer the Motion approaches to Theory; and when the Teeth are infinitely ſmall, the Motion will be theoretically true, with this Conſideration, the Power of the Lever is here introduced, acting from the Circumference of a perfect poliſhed Circle; whereby the Index, paſſing over the whole Arch of the Inſtrument, cauſes only a ſmall Motion of the Object Glaſs, the relative Motions of which are given in the Table annexed, and by which we are enabled to meaſure an Angle with all the Accuracy the Nature of Viſion will admit. The Motions being likewiſe adapted ſo as to exclude all Danger of Friction.

The Inſtrument, beſides this new Property, is a compleat *Hadley's* Quadrant improved by a Mode of Anticipation, the Glaſs being contrived to move without the Index, and the Index without the Glaſs, by which, in the Adjuſtment preparatory to Obſervation, we meaſure any Angle not unlimited by the Nature of Reflection, and which is more particularly explained in the following

DESCRIPTION.

Fig. I. Repreſents the whole Inſtrument, with as much of its particular Apparatus, as could be conveniently ſhewn at one View; it differs in many Reſpects from *Hadley's*; in the latter the Diviſions on the Limb are from M towards L; in this they are in the contrary Order, and are but half the Number in the ſame Space; T and *t*, are two ſight Vanes, to ſuit two Situations of the Object Glaſs, both directed to the ſame Horizon, or Eye Glaſs E; they are each adapted to receive a ſmall Teleſcope, with which we obſerve the Contacts more accurately; D and *d* are the dark Glaſſes. On the Center C of the Index, where the Index Glaſs of *Hadley's* Quadrant is uſually fixed, is a Braſs or Bell-metal Semicircle, about $\frac{1}{8}$ of an Inch thick (which is alſo the Subſtance of the other Plates hereafter to be deſcribed) its Diameter in the preſent Inſtrument two Inches. This Plate, in the

firſt Inſtance, was a whole Circle turned in a Lath, to render its perpendicular Circumference perfectly ſmooth, but was worked off afterwards to the Shape of the Index Head. It is ſcrewed faſt to the Index in ſuch a Manner, that the Axis of the Index is a Tangent to it. The Head of the Inſtrument does not terminate like *Hadley's*, but is continued in a right Line from the 0 Diviſion on the Limb, through the Center C to A. On the upper Center *a* are two circular Braſs Plates, which may revolve concentrically together, or ſeparately round, or upon, their common Center *a*. The under Plate which is repreſented by Fig. II, hath a continued Part *p. m.* the Surface, whereof which is perpendicular to the Plane of the Inſtrument, and which is acted againſt by the ſemicircular Piece on the lower Center, is directly ſtraight with the right Line *a. p. m.* The upper Plate, Fig. III, hath a Grove S, whereby it may be confined to the Inſtrument itſelf, or ſuffered to paſs freely by the Screw S. It has alſo another Grove *r*, whereby it may be confined to the under Plate, or ſuffered to paſs freely by the Screw *r*. O is the Object

Glaſs, or large Field Speculum, common to all reflecting Sea Quadrants, fixed with its adjuſting Screws upon the upper Plate. *f* is a Feather Spring, checked by the Screw *s*, when counter-acting the continued Part *p. m.* of the under Plate.

Fig. IV. Is a circular Braſs Plate, like the former, but has no continued Part; *i. i. i.* are Perforations agreeable to the three Pillars *i. i. i.* on the Head of the Index, where it is ſcrewed on faſt, not to be removed; its Center C is a Nut, to admit a Screw, to faſten the upper Plate, carrying the Object Glaſs, which may be removed here occaſionally, to perform their Offices in the ſame Manner as have been deſcribed on the upper Center. The Diſtance between the two Centers is four Inches, and here it may be proper to take Notice, *If it ſhould be ſuggeſted, that, on Account of the Nicety required, in giving the two Centers their true Poſition, the Inſtrument may not poſſeſs the pretended Certainty from a too great Dependence on the Artiſt*; That every ſeeming Difficulty of that Nature is

obviated by a certain and exact Method, perfectly conformable to the whole Theory.

Having thus deſcribed the Inſtrument, we may next go about to explain its Theory and Manner of Operation, in Reſpect to the upper Center.

The Object Glaſs being adjuſted and brought parallel with the Horizon Glaſs, if the Index be drawn from L towards M, the ſemicircular Piece which is on the Center of the Index, will act againſt the continued Part *p. m.* of the under Braſs Plate, and move it in the Direction of the Occult Arch; thus will the Object Glaſs O be altered with Reſpect to its Poſition with the ſmall Glaſs E. The Inclination of the two Glaſs Planes is juſt half the angular Diſtance. But to determine the Quantity of Motion, we muſt conſult the following Theory.

Since the Diſtance between the two Centers *a* and *C*, and alſo the Radius of the Braſs ſemicircular Piece on the Center of the Index, are conſtant Quantities, and the Motion

Table 1.

Index	Angular Distance				Differences				Diff. of Diff.
	°	′	″	10ths	°	′	″	10ths	
1	0	0	15	6	0	0	15	6	32.7
2		1	3	2		0	47	6	
3		2	23	4		1	20	2	
4		4	15	6		1	52	2	
5		6	41	2		2	25	6	
6		9	40	2		2	59		34.2
7		13	13			3	32	8	
8		17	20			4	7		
9		22	1	4		4	41	4	
10		27	17	8		5	16	4	
11		33	9	7		5	51	9	36.2
12		39	37	8		6	28	1	
13		46	42	7		7	4	9	
14		54	24	2		7	41	5	
15	1	2	41	8		8	17	6	
16		11	36	6		8	54	8	38
17		21	9	2		9	32	6	
18		31	19	8		10	10	6	
19		42	8	4		10	48	6	
20		53	35	8		11	27	4	
21	2	5	42	2		12	6	4	39.6
22		18	27	6		12	45	4	
23		31	52	4		13	24	8	
24		45	57			14	4	6	
25	3	0	41	8		14	44	8	
26		16	6	6		15	24	8	40.8
27		32	11	4		16	4	8	
28		48	56	4		16	45		
29	4	6	23			17	26	6	
30		24	30	6		18	7	6	
31		43	18	6		18	48		40.9
32	5	2	48			19	29	4	
33		22	58	4		20	10	4	
34		43	50	2		20	51	8	
35	6	5	23			21	32	8	
36		27	37	2		22	14	2	41
37		50	32	4		22	55	2	
38	7	14	8	8		23	36	4	
39		38	26	4		24	17	6	
40	8	3	24	6		24	58	2	
41		29	3	4		25	38	8	41
42		55	23			26	19	6	
43	9	22	23			27	0		
44		50	2	8		27	39	8	
45	10	18	21	8		28	19		

Table II.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
2	0 4																																																										
3	0 6	0 9																																																								
4	0 8	0 12	0 16																																																						
5	0 10	0 15	0 20	0 25																																																			
6	0 12	0 18	0 24	0 30	0 36																																																	
7	0 14	0 21	0 28	0 35	0 42	0 49																																																
8	0 16	0 24	0 32	0 40	0 48	0 56	1 4																																														
9	0 18	0 27	0 36	0 45	0 54	1 3	1 12	1 21																																												
10	0 20	0 30	0 40	0 50	1 0	1 10	1 20	1 30	1 40																																										
11	0 22	0 33	0 44	0 55	1 6	1 17	1 28	1 39	1 50	2 1																																								
12	0 24	0 36	0 48	1 0	1 12	1 24	1 36	1 48	2 0	2 12	2 24																																							
13	0 26	0 39	0 52	1 5	1 18	1 31	1 44	1 57	2 10	2 23	2 36	2 49																																						
14	0 28	0 42	0 56	1 10	1 24	1 38	1 52	2 6	2 20	2 34	2 48	3 2	3 16																																					
15	0 30	0 45	1 0	1 15	1 30	1 45	2 0	2 15	2 30	2 45	3 0	3 15	3 30	3 45																																				
16	0 32	0 48	1 4	1 20	1 36	1 52	2 8	2 24	2 40	2 56	3 12	3 28	3 44	4 0	4 16																																			
17	0 34	0 51	1 8	1 25	1 42	1 59	2 16	2 33	2 50	3 7	3 24	3 41	3 58	4 15	4 32	4 49																																		
18	0 36	0 54	1 12	1 30	1 48	2 6	2 24	2 42	3 0	3 18	3 36	3 54	4 12	4 30	4 48	5 6	5 24																																	
19	0 38	0 57	1 16	1 35	1 54	2 13	2 32	2 51	3 10	3 29	3 48	4 7	4 26	4 45	5 4	5 23	5 42	6 1																																
20	0 40	1 0	1 20	1 40	2 0	2 20	2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40																															
21	0 42	1 3	1 24	1 45	2 6	2 27	2 48	3 9	3 30	3 51	4 12	4 33	4 54	5 15	5 36	5 57	6 18	6 39	7 0	7 21																														
22	0 44	1 6	1 28	1 50	2 12	2 34	2 56	3 18	3 40	4 2	4 24	4 46	5 8	5 30	5 52	6 14	6 36	6 58	7 20	7 42	8 4																													
23	0 46	1 9	1 32	1 55	2 18	2 41	3 4	3 27	3 50	4 13	4 36	4 59	5 22	5 45	6 8	6 31	6 54	7 17	7 40	8 3	8 26	8 49																												
24	0 48	1 12	1 36	2 0	2 24	2 48	3 12	3 36	4 0	4 24	4 48	5 12	5 36	6 0	6 24	6 48	7 12	7 36	8 0	8 24	8 48	9 12	9 36																											
25	0 50	1 15	1 40	2 5	2 30	2 55	3 20	3 45	4 10	4 35	5 0	5 25	5 50	6 15	6 40	7 5	7 30	7 55	8 20	8 45	9 10	9 35	10 0	10 25																										
26	0 52	1 18	1 44	2 10	2 36	3 2	3 28	3 54	4 20	4 46	5 12	5 38	6 4	6 30	6 56	7 22	7 48	8 14	8 40	9 6	9 32	9 58	10 24	10 50	11 16																									
27	0 54	1 21	1 48	2 15	2 42	3 9	3 36	4 3	4 30	4 57	5 24	5 51	6 18	6 45	7 12	7 39	8 6	8 33	9 0	9 27	9 54	10 21	10 48	11 15	11 42	12 9																								
28	0 56	1 24	1 52	2 20	2 48	3 16	3 44	4 12	4 40	5 8	5 36	6 4	6 32	7 0	7 28	7 56	8 24	8 52	9 20	9 48	10 16	10 44	11 12	11 40	12 8	12 36	13 4																							
29	0 58	1 27	1 56	2 25	2 54	3 23	3 52	4 21	4 50	5 19	5 48	6 17	6 46	7 15	7 44	8 13	8 42	9 11	9 40	10 9	10 38	11 7	11 36	12 5	12 34	13 3	13 32	14 1																						
30	1 0	1 30	2 0	2 30	3 0	3 30	4 0	4 30	5 0	5 30	6 0	6 30	7 0	7 30	8 0	8 30	9 0	9 30	10 0	10 30	11 0	11 30	12 0	12 30	13 0	13 30	14 0	14 30	15 0																															
31	1 2	1 33	2 4	2 35	3 6	3 37	4 8	4 39	5 10	5 41	6 12	6 43	7 14	7 45	8 16	8 47	9 18	9 49	10 20	10 51	11 22	11 53	12 24	12 55	13 26	13 57	14 28	14 59	15 30	16 1																														
32	1 4	1 36	2 8	2 40	3 12	3 44	4 16	4 48	5 20	5 52	6 24	6 56	7 28	8 0	8 32	9 4	9 36	10 8	10 40	11 12	11 44	12 16	12 48	13 20	13 52	14 24	14 56	15 28	16 0	16 32	17 4																												
33	1 6	1 39	2 12	2 45	3 18	3 51	4 24	4 57	5 30	6 3	6 36	7 9	7 42	8 15	8 48	9 21	9 54	10 27	11 0	11 33	12 6	12 39	13 12	13 45	14 18	14 51	15 24	15 57	16 30	17 3	17 36	18 9																											
34	1 8	1 42	2 16	2 50	3 24	3 58	4 32	5 6	5 40	6 14	6 48	7 22	7 56	8 30	9 4	9 38	10 12	10 46	11 20	11 54	12 28	13 2	13 36	14 10	14 44	15 18	15 52	16 26	17 0	17 34	18 8	18 42	19 16																										
35	1 10	1 45	2 20	2 55	3 30	4 5	4 40	5 15	5 50	6 25	7 0	7 35	8 10	8 45	9 20	9 55	10 30	11 5	11 40	12 15	12 50	13 25	14 0	14 35	15 10	15 45	16 20	16 55	17 30	18 5	18 40	19 15	19 50	20 25																									
36	1 12	1 48	2 24	3 0	3 36	4 12	4 48	5 24	6 0	6 36	7 12	7 48	8 24	9 0	9 36	10 12	10 48	11 24	12 0	12 36	13 12	13 48	14 24	15 0	15 36	16 12	16 48	17 24	18 0	18 36	19 12	19 48	20 24	21 0	21 36																								
37	1 14	1 51	2 28	3 5	3 42	4 19	4 56	5 33	6 10	6 47	7 24	8 1	8 38	9 15	9 52	10 29	11 6	11 43	12 20	12 57	13 34	14 11	14 48	15 25	16 2	16 39	17 16	17 53	18 30	19 7	19 44	20 21	20 58	21 35	22 12	22 49																							
38	1 16	1 54	2 32	3 10	3 48	4 26	5 4	5 42	6 20	6 58	7 36	8 14	8 52	9 30	10 8	10 46	11 24	12 2	12 40	13 18	13 56	14 34	15 12	15 50	16 28	17 6	17 44	18 22	19 0	19 38	20 16	20 54	21 32	22 10	22 48	23 26	24 4																						
39	1 18	1 57	2 36	3 15	3 54	4 33	5 12	5 51	6 30	7 9	7 48	8 27	9 6	9 45	10 24	11 3	11 42	12 21	13 0	13 39	14 18	14 57	15 36	16 15	16 54	17 33	18 12	18 51	19 30	20 9	20 48	21 27	22 6	22 45	23 24	24 3	24 42	25 21																					
40	1 20	2 0	2 40	3 20	4 0	4 40	5 20	6 0	6 40	7 20	8 0	8 40	9 20	10 0	10 40	11 20	12 0	12 40	13 20	14 0	14 40	15 20	16 0	16 40	17 20	18 0	18 40	19 20	20 0	20 40	21 20	22 0	22 40	23 20	24 0	24 40	25 20	26 0	26 40																				
41	1 22	2 3	2 44	3 25	4 6	4 47	5 28	6 9	6 50	7 31	8 12	8 53	9 34	10 15	10 56	11 37	12 18	12 59	13 40	14 21	15 2	15 43	16 24	17 5	17 46	18 27	19 8	19 49	20 30	21 11	21 52	22 33	23 14	23 55	24 36	25 17	25 58	26 39	27 20	28 1																			
42	1 24	2 6	2 48	3 30	4 12	4 54	5 36	6 18	7 0	7 42	8 24	9 6	9 48	10 30	11 12	11 54	12 36	13 18	14 0	14 42	15 24	16 6	16 48	17 30	18 12	18 54	19 36	20 18	21 0	21 42	22 24	23 6	23 48	24 30	25 12	25 54	26 36	27 18	28 0	28 42	29 24																		
43	1 26	2 9	2 52	3 35	4 18	5 1	5 44	6 27	7 10	7 53	8 36	9 19	10 2	10 45	11 28	12 11	12 54	13 37	14 20	15 3	15 46	16 29	17 12	17 55	18 38	19 21	20 4	20 47	21 30	22 13	22 56	23 39	24 22	25 5	25 48	26 31	27 14	27 57	28 40	29 23	30 6	30 49																	
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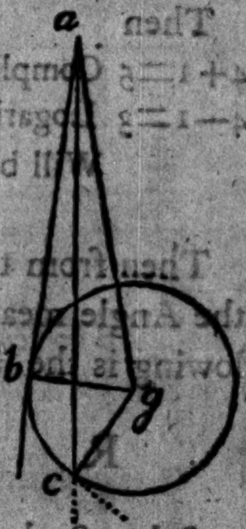
45	1 30	2 15	3 0	3 45	4 30	5 15	6 0	6 45	7 30	8 15	9 0	9 45	10 30	11 15	12 0	12 45	13 30	14 15	15 0	15 45	16 30	17 15	18 0	18 45	19 30	20 15	21 0	21 45	22 30	23 15	24 0	24 45	25 30	26 15	27 0	27 45	28 30	29 15	30 0	30 45	31 30	32 15	33 0	33 45															
46	1 32	2 18	3 4	3 50	4 36	5 22	6 8	6 54	7 40	8 26	9 12	9 58	10 44	11 30	12 16	13 2	13 48	14 34	15 20	16 6	16 52	17 38	18 24	19 10	19 56	20 42	21 28	22 14	23 0	23 46	24 32	25 18	26 4	26 50	27 36	28 22	29 8	29 54	30 40	31 26	32 12	32 58	33 44	34 30	35 16														
47	1 34	2 21	3 8	3 55	4 42	5 29	6 16	7 3	7 50	8 37	9 24	10 11	10 58	11 45	12 32	13 19	14 6	14 53	15 40	16 27	17 14	18 1	18 48	19 35	20 22	21 9	21 56	22 43	23 30	24 17	25 4	25 51	26 38	27 25	28 12	28 59	29 46	30 33	31 20	32 7	32 54	33 41	34 28	35 15	36 2	36 49													
48	1 36	2 24	3 12	4 0	4 48	5 36	6 24	7 12	8 0	8 48	9 36	10 24	11 12	12 0	12 48	13 36	14 24	15 12	16 0	16 48	17 36	18 24	19 12	20 0	20 48	21 36	22 24	23 12	24 0	24 48	25 36	26 24	27 12	28 0	28 48	29 36	30 24	31 12	32 0	32 48	33 36	34 24	35 12	36 0	36 48	37 36	38 24												
49	1 38	2 27	3 16	4 5	4 54	5 43	6 32	7 21	8 10	8 59	9 48	10 37	11 26	12 15	13 4	13 53	14 42	15 31	16 20	17 9	17 58	18 47	19 36	20 25	21 14	22 3	22 52	23 41	24 30	25 19	26 8	26 57	27 46	28 35	29 24	30 13	31 2	31 51	32 40	33 29	34 18	35 7	35 56	36 45	37 34	38 23	39 12	40 1											
50	1 40	2 30	3 20	4 10	5 0	5 50	6 40	7 30	8 20	9 10	10 0	10 50	11 40	12 30	13 20	14 10	15 0	15 50	16 40	17 30	18 20	19 10	20 0	20 50	21 40	22 30	23 20	24 10	25 0	25 50	26 40	27 30	28 20	29 10	30 0	30 50	31 40	32 30	33 20	34 10	35 0	35 50	36 40	37 30	38 20	39 10	40 0	40 50	41 40										
51	1 42	2 33	3 24	4 15	5 6	5 57	6 48	7 39	8 30	9 21	10 12	11 3	11 54	12 45	13 36	14 27	15 18	16 9	17 0	17 51	18 42	19 33	20 24	21 15	22 6	22 57	23 48	24 39	25 30	26 21	27 12	28 3	28 54	29 45	30 36	31 27	32 18	33 9	34 0	34 51	35 42	36 33	37 24	38 15	39 6	39 57	40 48	41 39	42 30	43 21									
52	1 44	2 36	3 28	4 20	5 12	6 4	6 56	7 48	8 40	9 32	10 24	11 16	12 8	13 0	13 52	14 44	15 36	16 28	17 20	18 12	19 4	19 56	20 48	21 40	22 32	23 24	24 16	25 8	26 0	26 52	27 44	28 36	29 28	30 20	31 12	32 4	32 56	33 48	34 40	35 32	36 24	37 16	38 8	39 0	39 52	40 44	41 36	42 28	43 20	44 12	45 4								
53	1 46	2 39	3 32	4 25	5 18	6 11	7 4	7 57	8 50	9 43	10 36	11 29	12 22	13 15	14 8	15 1	15 54	16 47	17 40	18 33	19 26	20 19	21 12	22 5	22 58	23 51	24 44	25 37	26 30	27 23	28 16	29 9	30 2	30 55	31 48	32 41	33 34	34 27	35 20	36 13	37 6	37 59	38 52	39 45	40 38	41 31	42 24	43 17	44 10	45 3	45 56	46 49							
54	1 48	2 42	3 36	4 30	5 24	6 18	7 12	8 6	9 0	9 54	10 48	11 42	12 36	13 30	14 24	15 18	16 12	17 6	18 0	18 54	19 48	20 42	21 36	22 30	23 24	24 18	25 12	26 6	27 0	27 54	28 48	29 42	30 36	31 30	32 24	33 18	34 12	35 6	36 0	36 54	37 48	38 42	39 36	40 30	41 24	42 18	43 12	44 6	45 0	45 54	46 48	47 42	48 36						
55	1 50	2 45	3 40	4 35	5 30	6 25	7 20	8 15	9 10	10 5	11 0	11 55	12 50	13 45	14 40	15 35	16 30	17 25	18 20	19 15	20 10	21 5	22 0	22 55	23 50	24 45	25 40	26 35	27 30	28 25	29 20	30 15	31 10	32 5	33 0	33 55	34 50	35 45	36 40	37 35	38 30	39 25	40 20	41 15	42 10	43 5	44 0	44 55	45 50	46 45	47 40	48 35	49 30	50 25					
56	1 52	2 48	3 44	4 40	5 36	6 32	7 28	8 24	9 20	10 16	11 12	12 8	13 4	14 0	14 56	15 52	16 48	17 44	18 40	19 36	20 32	21 28	22 24	23 20	24 16	25 12	26 8	27 4	28 0	28 56	29 52	30 48	31 44	32 40	33 36	34 32	35 28	36 24	37 20	38 16	39 12	40 8	41 4	42 0	42 56	43 52	44 48	45 44	46 40	47 36	48 32	49 28	50 24	51 20	52 16				
57	1 54	2 51	3 48	4 45	5 42	6 39	7 36	8 33	9 30	10 27	11 24	12 21	13 18	14 15	15 12	16 9	17 6	18 3	19 0	19 57	20 54	21 51	22 48	23 45	24 42	25 39	26 36	27 33	28 30	29 27	30 24	31 21	32 18	33 15	34 12	35 9	36 6	37 3	38 0	38 57	39 54	40 51	41 48	42 45	43 42	44 39	45 36	46 33	47 30	48 27	49 24	50 21	51 18	52 15	53 12	54 9			
58	1 56	2 54	3 52	4 50	5 48	6 46	7 44	8 42	9 40	10 38	11 36	12 34	13 32	14 30	15 28	16 26	17 24	18 22	19 20	20 18	21 16	22 14	23 12	24 10	25 8	26 6	27 4	28 2	29 0	29 58	30 56	31 54	32 52	33 50	34 48	35 46	36 44	37 42	38 40	39 38	40 36	41 34	42 32	43 30	44 28	45 26	46 24	47 22	48 20	49 18	50 16	51 14	52 12	53 10	54 8	55 6	56 4		
59	1 58	2 57	3 56	4 55	5 54	6 53	7 52	8 51	9 50	10 49	11 48	12 47	13 46	14 45	15 44	16 43	17 42	18 41	19 40	20 39	21 38	22 37	23 36	24 35	25 34	26 33	27 32	28 31	29 30	30 29	31 28	32 27	33 26	34 25	35 24	36 23	37 22	38 21	39 20	40 19	41 18	42 17	43 16	44 15	45 14	46 13	47 12	48 11	49 10	50 9	51 8	52 7	53 6	54 5	55 4	56 3	57 2	58 1	
60	2 0	3 0	4 0	5 0	6 0	7 0	8 0	9 0	10 0	11 0	12 0	13 0	14 0	15 0	16 0	17 0	18 0	19 0	20 0	21 0	22 0	23 0	24 0	25 0	26 0	27 0	28 0	29 0	30 0	31 0	32 0	33 0	34 0	35 0	36 0	37 0	38 0	39 0	40 0	41 0	42 0	43 0	44 0	45 0	46 0	47 0	48 0	49 0	50 0	51 0	52 0	53 0	54 0	55 0	56 0	57 0	58 0	59 0	60

Motion of the ſemicircular Piece is determined by the Index on the Limb, we have

Given $\left\{\begin{array}{l}\text{the Side } aC \\ \text{the Side } Cg \\ \angle aCg \text{———}\end{array}\right\}$

to determine
the
$\angle Cag$

In every Moment of the Motion.



The Proportion is

$aC+Cg : aC-Cg :: \text{Tang. } \tfrac{1}{2}\overline{Cag+Cga}$
$: \text{Tang. } \tfrac{1}{2}\overline{Cga-Cag}$

Then $\tfrac{1}{2}\overline{Cag+Cga} - \tfrac{1}{2}\overline{Cga-Cag} = \angle Cag$

Now $\triangle abg$ is a right $\angle$ $\triangle$.

And becauſe $bg=Cg$. and ag is common to both; we have Sine aCg : Radius :: Sine Cag : Sine gab.

Then $\angle gab - \angle Cag = \angle Cab =$ the Inclination of the two Glaſs Planes.

Now in the preſent Inſtrument, let the Diſtance between the Centers a and C be four Inches, and the Radius of the Braſs Semicircle on the Index be one Inch——

Then

4+1=5	Compliment Arithmetical	8,3010300
4—1=3	Logarithm — —	1,4771213
	Will be a conſtant Logar.	9,7781513

Then from the above Proportions to find the Angle meaſured by Reflection, the following is the

R U L E.

Subtract the Index Angle from 90 and the Remainder from 180. Add the Logar. Tangent of half the laſt Remainder to the conſtant Logarithm. From the laſt Remainder ſubtract the Tangent Angle of this Sum, to the Logarithm Sine of this Difference; add the Compliment arithmetical of the Logarithm Sine of the firſt Remainder. From the ſinical Angle of this Aggregate, ſubtract the Difference laſt found, the Remainder will be the true Inclination of the two Glaſs Planes, which being doubled, gives the true angular Diſtance.

EXAMPLE

EXAMPLE.

Suppoſe the Angle ſhewn by the Index

```
                              20° 0′ 0″. 0
                              70
                              110
                              55  0  0   Tangent          10, 1547732
                                         Conſtant Log.     9, 7781513
                              40 35 34. 4Tangent           9, 9329245
9, 3958680 ——Sine——           14 24 25. 6                        055
0, 0270142 Co. Ar. S. 70                                   426)1900(4. 4
9, 4228822 ——Sine——           15 21 13. 6                      1704
     547                       0 56 48                         1960
  767)2750(3. 6                1 53 36   Angular Diſtance
     2301
     4490
```

In this Manner Table I. for all Angles was conſtructed.

To diſcover the Advantages the Inſtrument has of the common Quadrants, let it be conſidered, that while the Index is moving from L towards M, with an equable Motion, the Inclination of the two Glaſs Planes is altering much ſlower, and the Degrees of Perfection is in the Ratio of their Velocities directly.

Wherefore, at the Concluſion of an Obſervation, the nearer the Index is to the 0 Diviſion, on the Limb, the greater is the Degree of Correction.

COMPARATIVE EXAMPLES.

Suppoſe the Angle ſhewn by the Index was
22 0 0 = by the Tables 2 18 27.6

But repeating the Obſervation it appears	22 0 30	2 18 34
The Differences are	0 0 30	0 0 6.4
But by the common Quadrants from the Nature of Reflection the angular Error is double.	0 1 0	

Which is nearly ten Times as great as in the new Inſtrument

Suppoſe in meaſuring the Sun's Diameter, the Angle ſhewn by the Index was

	10 50 0	=by the Tables	0 32 6, 1
And by repeating	10 50 30±		0 32 9
The Differences are	0 0 30		0 0 2. 9
Which doubled for Reflection	0 1 0		

An Error near Twenty-one Times as great as by the New Inſtrument.

Thus far we have conſidered the Operation of the Inſtrument only during the Motion of the Index from L to M; we have now to obſerve that at M the Motion is finiſhed, and the greateſt Angle we can meaſure is 10 18 1. 8; but if we would meaſure a larger Angle, the Plates which move Concentrically on the upper Center *a*, are contrived to anticipate any Space in adjuſting the Object Glaſs, *viz.* let the Screw S be made faſt, and looſen the Screw *r*; in this Situation of the Screws, adjuſt the Glaſſes by the Modes in Practice, bring forward the Index to any Quantity on the Limb, and make it faſt; alſo make faſt the

Screw *r*, and looſen the Screw S, then may the Index return to the 0 Diviſion, but the Object Glaſs will remain; make faſt the Screw S, and looſen the Screw *r*, and the Inſtrumnet is ready for Obſervation, or for repeating this Operation, if neceſſary, always obſerving to add the Quantity of Adjuſtment to the next Obſervation, and the Sum will be the true angular Diſtance.

After this Manner we may meaſure Angles as correct as we pleaſe, for if we gueſs at but nearly the expected Quantity, we may adjuſt the Inſtrument accordingly.

EXAMPLE.

Suppoſe in meaſuring an Angle the Index was at 44 0 0 = 9 50 2.8

But in repeating the Obſervation, it is read off } 44 0 30 = 9 50 16.4

The Error of Obſervation is 0 0 13.6

But by the Method of Anticipation, move the Index to 27 0 0 = 3 32 11.4

and repeat it 27 0 0 = 3 32 11.4

The Obſervation would be } 23 59 0 = 2 45 43

9 50 5.8

Or repeating perhaps it would be	23 59 30=2 45 50. 2
Which added to the two Anticipations is — — —	9 50 13
Wherefore the Limits, inſtead of 13. 6, is only — —	0 0 7. 2

In Order that the Obſerver may well underſtand how to turn the Method of Anticipation to Advantage, in the laſt particular, he muſt take Notice, that the Quantity to be anticipated depends on the Circumſtances of the Quantity to be meaſured, and that without ſome Preconſideration in that Reſpect, his laſt Reſult may be but little better, and ſometimes more erroneous than the firſt, for the Degree of Correction is not determined by the Quantity deduced from the laſt Poſition of the Index, but by the Sum of all the Errors liable to be commited in each Time of reading off. To elucidate this Obſervation, let us examine the laſt Example.

The Error liable to be commited in reading off, from the firſt Anticipation, appears by the Tables to be } 27=8
and by repeating the ſame — — 8
and in the final Obſervation — 23 59= 7
23

but the Error of Obſervation at firſt was 13.6
Which is but little more than half ſo great as the Error above.

Wherefore to apply the Method of Anticipation, to get a greater Correction in that Example, we ſhould conſider that, firſt, the Stops which check the Index at each End of the Arch, are placed true to their Situation; and, ſecondly, that the Quantity between 9 and 10 is nearly as much as the whole Arch will meaſure; then if we ſet the Index in Advance a little Way, we ſhall read twice towards the firſt End of the Arch, and conſequently more correctly, *viz.*

Suppoſe the Index put forwards to
12=0 39 37.8
Adjuſt the Inſtrument the ſame as if it was at the 0 Diviſion.

Move the Index to 45, and the Quantity anticipated will be the Reſidue from 10 18 21.8 } = 9 38 44

Then the Obſervation will be 6 32=0 11 26
9 50 10

In the firſt Situation the Error would be = 3.2
In the laſt —— —— —— 1.7
The whole is only —— —— —— 4.9

Of adjuſting and uſing the Inſtrument.

It would be a fruitleſs Buſineſs to conſtruct an Inſtrument to perform with Accuracy, if the ſtricteſt Attention was not afterwards paid to the Adjuſtment of the ſeveral Glaſſes; and as they are liable to be injured in this Reſpect, ſo as ſcarcely to preſerve their true Poſitions for two Days together, it is a Matter which calls for the utmoſt Diligence in the Obſerver to acquire an eaſy and certain Method of diſcovering the Faults, and making the neceſſary Rectifications. The firſt Thing to be attended to, is to ſet the Horizon Glaſs perpendicular to the Plane of the Inſtrument, which is performed as follows: Hold the Inſtrument with its Plane perpendicular to the Horizon, and look over backwards into the Glaſs and beyond it. If the Limb of the Inſtrument appears in a right Line with its Reflection, the Glaſs is upright; but if it does not appear ſo, looſen or tighten the little Screw on the Foot of the Glaſs, until it is adjuſted; then with the Inſtrument, as in taking an Altitude, look through the ſight Vane, or Teleſcope, at ſome diſtant Object, with the Index fixed in any intended Situation; the Finger Screws S and *r* being both looſe, turn the Glaſs about till the ſame Object appears nearly in the ſame Part of the Horizon Glaſs; next hold it in a horizontal Poſition, and adjuſt the Object Glaſs with the Screws which are behind and before, on the Foot of it, till the Object and its Reflection is ſeen in the ſame horizontal Line. Laſtly, with the Inſtrument upright, turn the Tangent Screw belonging to the Horizon Glaſs, at the back of the Inſtrument, until there is a perfect Coincidence of the Object and its Reflection that Way, and the Adjuſtments are compleat.

The beſt Object for the Purpoſe, is the Sun's Limb, viewed by Help of a dark Glaſs ſcrewed on the Teleſcope; Care being taken to add the Quantity of the Sun's Diameter to the ſucceeding Obſervations, or ſubtract it therefrom, according to the Limb in Contact with its reflected Image.

Explanation and Uſe of the Tables.

The ſmall Table, Nº I. conſiſts of four Columns; the firſt indicating the Poſition of the Index at every whole Degree; the ſecond, the reſpective angular Diſtance derived from the Motion of the Object Glaſs; the third exhibits the increaſing Differences of the angular Diſtance; and, in the laſt Column, are the mean ſecond Differences to every five Degrees.

The Quantity being given for every whole Degree in the ſecond Column, the Reſidue for intermediate Minutes and Seconds might be found as in the common Way, by Proportion from the Sums contained in the third and fourth; but as that Method is attended with ſome Trouble, the Operation will be rendered eaſy and true by Help of the Table Nº II, which ſhews the Quantity ariſing from the Multiplication of one Sexageſimal, by another of the ſame, or a lower Denomination, as Degrees multiplied by Minutes, or Minutes multiplied by Seconds; and the Reſult is given in Minutes and Seconds, or Seconds and Thirds; their Uſes will be more clearly underſtood from the following

EXAMPLE.

Suppoſe the Quantity ſhewn by the Index in an Obſervation was 25 10

From Table I. 25 is 3 0 41. 8 | 14 44. 8 | 40

in the large Table ſeek 40 down the left hand Column, & under 10 is	— — — 6. 7
	14 51. 5
Then under 10 againſt 14 is	0 2 20
And under 10 againſt 51. 5 is	0 0 8. 6
The Sum is the true angular Diſtance, agreeing with 25,10	3 3 10. 4

The Table N° II. will be found of infinite Service in all aſtronomical Calculations, in determining the intermediate Quantities where the Differences are given, particularly in uſing *Mayer's* Solar and Lunar Tables.

Of

OF THE

LONGITUDE.

THE Method of finding the Longitude by obſerving the Moon's Diſtance from the Sun or a fixed Star is now the moſt approved, and as it is growing more and more familiar from the continual Improvements in the Mode of Calculation, there is ſcarcely a Doubt but it will be univerſally practiſed on Board of all large Ships, where there are ſeveral good Inſtruments and Obſervers. But as long Voyages are now frequently undertaken by Ships of 150 or 200 Tons Burthen, where there is ſeldom more than one or two at the moſt capable of obſerving correctly; it is preſumed, the Method hereafter laid down of aſcertaining it from the Moon's Altitude only, will not be unacceptable to the Mariner, as it is here attended with Tables for reducing the Apparent, to the true Altitude, calculated for that Purpoſe to a greater Nicety and Perfection than any hitherto made public.

Dr. *Maſkelyne*, in the Solar and Lunar Tables, Page 120, has given the following very convenient Formula, for finding the Quantity of Refraction at any Altitude, and at any State of the Barometer and Thermometer, and recommends the Uſe thereof in all low Altitudes, in Preference to taking it by Proportion from the Tables which are calculated to Degrees only, *viz.*

Let the apparent Zenith Diſtance $= d$
The Height of the Baro. in *Engliſh* Inches $= b$
And the Height of *Fahrenheit*'s Therm. $= p$

Find an Angle w, whoſe Tangent is

$$\frac{\left(1+\frac{p}{400}-\frac{p}{60000}\right)^{\frac{1}{2}}}{17{,}143\ \text{Coſ.}\ d}$$

and the Refraction is

$$\frac{74{,}408\ b \times \text{Sine}\ d \times \text{Tang.}\ \frac{1}{2}\ w.}{\left(1+\frac{p}{400}-\frac{p}{60000}\right)^{\frac{3}{2}}}$$

Prob. I. To find the Quantity of Refraction at the mean State of the Air, *viz.* Height of the Barometer 29 6 Inches, and Height of the Thermometer 50. ſubſtitute the Values for p and b, and the Expreſſions will be

$$\frac{\left(1+\frac{50}{400}-\frac{50}{60000}\right)^{\frac{1}{2}}}{17{,}143 \text{ Coſ. } d}$$

$$\text{That is } \frac{1{,}1241666^{\frac{1}{2}}}{17{,}143} \div \text{S. Altitude} = \text{Tang. } w.$$

$$\text{And } \frac{74{,}408\times 29.6}{\left(1+\frac{50}{400}-\frac{50}{60000}\right)^{\frac{3}{2}}} \times \text{S. } d \times \text{T. } \tfrac{1}{2} w = \text{the Refr.}$$

$$\text{Then } \frac{1{,}1241666^{\frac{1}{2}}}{17{,}143} = \text{Log. } 8{,}79133 \text{ will be a conſtant Log. N° I.}$$

$$\text{And } \frac{74{,}408\times 29.6}{\left(1+\frac{50}{400}-\frac{50}{60000}\right)^{\frac{3}{2}}} = \text{Log. } 3{,}26667 \text{ will be a conſt. Log. N° II.}$$

From which we have the following

RULE

RULE.

From the Log. N° I. ſubtract the Log. Sine of the apparent Altitude, the Remainder is the Tangent of w.

To the Tangent of $\frac{1}{2}w$, add the Log. N° II. and the Log. Co-ſine of the Altitude, the Sum is the Log. of the Refraction in Seconds.

EXAMPLE.

For the Refraction at 7 42.

	8,79133	Log. N° I.
Sine 7 42———	9,12706	
24 46 42—T	9,66427	
12 23 21—T	9,34176	
	3,26667	Log. N° II.
Cos. 7 42———	9,99607	
	2,60450=402. 6=6 42. 6	

In the ſame Manner, ſubſtituting 70 for p, and 30 5 for b ſeparately, the Columns of Variation for the State of the Barometer and Thermometer, were compoſed in the following Table.

TABLE III. exhibiting the Quantity of Refraction at all Altitudes, at a mean State of the Air, *viz.* Barometer 29 6. Thermometer 50.

Apparent Altitude.	Mean Refraction	Variat. for Baro. $\frac{9}{10}$ In.	Variat. for Ther. 20	Apparent Altitude.	Mean Refraction	Variat. for Baro. $\frac{9}{10}$ In.	Variat. for Ther. 20
3 0	14 18	24	44. 5	3. 40	12 28		37. 6
1	14 15			41	12 25		
2	14 12			42	12 23		
3	14 9			43	12 20		
4	14 6			44	12 18		
5	14 3			45	12 16	20. 6	
6	14 0			46	12 13		
7	13 57			47	12 11		
8	13 54			48	12 8		
9	13 51			49	12 6		
10	13 48		42. 6	50	12 4		36. 2
11	13 45			51	12 1		
12	13 42			52	11 59		
13	13 39			53	11 57		
14	13 36			54	11 55		
15	13 33	22. 7		55	11 52		
16	13 31			56	11 50		
17	13 28			57	11 48		
18	13 25			58	11 46		
19	13 22			59	11 43		
20	13 20		40. 8	4. 0	11 41	19. 7	34. 9
21	13 17			1	11 39		
22	13 14			2	11 37		
23	13 11			3	11 35		
24	13 9			4	11 32		
25	13 6			5	11 30		
26	13 3			6	11 28		
27	13 1			7	11 26		
28	12 58			8	11 24		
29	12 55			9	11 22		
30	12 53	21. 6	39. 2	10	11 20		33. 7
31	12 50			11	11 18		
32	12 48			12	11 16		
33	12 45			13	11 14		
34	12 43			14	11 12		
35	12 40			15	11 10		
36	12 38			16	11 7		
37	12 35			17	11 5		
38	12 33			18	11 3		
39	12 30			19	11 1		

Apparent Altitude	Mean Refraction	Variat. for Baro. $\frac{1}{10}$ In.	Variat. for Ther. 20	Apparent Altitude	Mean Refraction	Variat. for Baro. $\frac{1}{10}$ In.	Variat. for Ther. 20
4. 20	11 0		32. 5	5 0	9 48	16. 6	28. 6
21	10 58			2	9 45		
22	10 56			4	9 42		
23	10 54			6	9 39		
24	10 52			8	9 36		
25	10 50			10	9 32		27. 7
26	10 48			12	9 29		
27	10 46			14	9 26		
28	10 44			16	9 23		
29	10 42			18	9 21		
30	10 40	18. 1	31. 4	20	9 18		26. 8
31	10 38			22	9 15		
32	10 37			24	9 12		
33	10 35			26	9 9		
34	10 33			28	9 6		
35	10 31			30	9 3		25 9.
36	10 29			32	9 0		
37	10 27			34	8 58		
38	10 26			36	8 55		
39	10 24			38	8 52		
40	10 22		30. 4	40	8 50		25. 1
41	10 20			42	8 47		
42	10 19			44	8 44		
43	10 17			46	8 42		
44	10 15			48	8 39		
45	10 13			50	8 37		24. 3
46	10 11			52	8 34		
47	10 10			54	8 32		
48	10 8			56	8 29		
49	10 6			58	8 27		
50	10 5		29. 5	6 0	8 24	14. 5	23. 5
51	10 3			2	8 22		
52	10 1			4	8 20		
53	10 0			6	8 17		
54	9 58			8	8 15		
55	9 56			10	8 12		
56	9 55			12	8 10		
57	9 53			14	8 8		
58	9 51			16	8 5		
59	9 50			18	8 3		

Apparent Altitude		Mean Refraction		Variat. for Baro. ${}_{1}{}^{9}{}_{0}$ In.	Variat. for Ther. 20	Apparent Altitude		Mean Refraction		Variat. for Baro. ${}_{1}{}^{9}{}_{0}$ In.	Variat. for Ther. 20
5	20	8	1			8	0	6	29	11 6	18 5
	22	7	59				3	6	27		
	24	7	56				6	6	25		
	26	7	54				9	6	23		
	28	7	52				12	6	20		
	30	7	50				15	6	18		
	32	7	48				18	6	16		
	34	7	46				21	6	14		
	36	7	44				24	6	12		
	38	7	42				27	6	10		
	40	7	40				30	6	8		
	42	7	38				35	6	6		
	44	7	36				36	6	4		
	46	7	34				39	6	2		
	48	7	32				42	6	0		
	50	7	30				45	5	58		
	52	7	28				48	5	56		
	54	7	26				51	5	54		
	56	7	24				54	5	52		
	58	7	22				57	5	50		
7	0	7	20	12 9	21	9	0	5	49	10 4	16 5
	3	7	17				3	5	47		
	6	7	14				6	5	45		
	9	7	11				9	5	43		
	12	7	9				12	5	41		
	15	7	6				15	5	40		
	18	7	3				18	5	38		
	21	7	1				21	5	36		
	24	6	58				24	5	35		
	27	6	56				27	5	33		
	30	6	53				30	5	31		
	33	6	51				33	5	30		
	36	6	48				36	5	28		
	39	6	46				39	5	26		
	42	6	43				42	5	25		
	45	6	41				45	5	23		
	48	6	38				48	5	22		
	51	6	36				51	5	20		
	54	6	34				54	5	19		
	57	6	31				57	5	17		

Apparent Altitude	Mean Refraction		Difference	Variat. for Baro. $_{1}{}^{9}{}_{0}$ In.		Variat. for Ther. 20		Apparent Altitude	Mean Refraction		Variat. for Baro. $_{1}{}^{9}{}_{0}$ In.		Variat. for Ther. 20	
10	5	15	28	9	3	14	9	50	0	48	1	4	2	2
11	4	47	23	8	6	13	7	51		47	1	4	2	1
12	4	24	20	7	9	12	5	52		45	1	3	2	0
13	4	4	18	7	3	11	5	53		43	1	3	2	0
14	3	46	15	6	8	10	6	54		42	1	2	1	9
15	3	31	14	6	3	9	9	55		40	1	2	1	8
16	3	17	12	5	9	9	3	56		39	1	2	1	7
17	3	5	11	5	6	8	7	57		37	1	1	1	7
18	2	54	9	5	3	8	2	58		36	1	1	1	6
19	2	45	9	5	0	7	7	59		34	1		1	5
20	2	36	8	4	7	7	3	60		33	1		1	5
21	2	28	7	4	4	6	9	61		32	1		1	4
22	2	21	7	4	2	6	6	62		31	0	9	1	4
23	2	14	6	4	0	6	3	63		29		9	1	3
24	2	8	6	3	7	5	9	64		28		8	1	3
25	2	2	5	3	5	5	6	65		27		8	1	2
26	1	57	5	3	4	5	3	66		26		8	1	2
27	1	52	4	3	2	5	1	67		24		7	1	1
28	1	48	5	3	1	4	9	68		23		7	1	0
29	1	43	4	3	0	4	7	69		22		7	1	0
30	1	39	4	2	9	4	5	70		21		6	0	9
31	1	35	3	2	8	4	3	71		20		6		9
32	1	32	4	2	7	4	2	72		19		6		8
33	1	28	3	2	6	4	1	73		18		5		8
34	1	25	3	2	5	3	9	74		17		5		7
35	1	22	3	2	5	3	8	75		15		5		7
36	1	19	3	2	4	3	7	76		14		4		6
37	1	16	3	2	3	3	5	77		13		4		6
38	1	13	2	2	3	3	4	78		12		4		5
39	1	11	3	2	2	3	2	79		11		3		5
40	1	8	2	2	1	3	1	80		10		3		4
41	1	6	2	2	0	3	0	81		9		3		4
42	1	4	3	2	0	2	9	82		8		2		3
43	1	1	2	1	9	2	8	83		7		2		3
44		59	2	1	8	2	7	84		6		2		2
45		57	2	1	7	2	6	85		5		2		2
46		55	2	1	6	2	5	86		4		1		2
47		53	1	1	6	2	4	87		3		1		1
48		52	2	1	5	2	3	88		2		1		1
49		50		1	5	2	2	89		1		0		0

TABLE IV.

From the Moon's Equatorial Parallax, or that given in the Ephemeris, to find the horizontal Parallax.

Latitude	′ 54		′ 56		′ 58		′ 60	
0	0	0	0	0	0	0	0	0
6	0	2	0	2	0	2	0	2
12	0	6	0	6	0	7	0	7
18	1	4	1	4	1	5	1	5
24	2	3	2	3	2	5	2	6
30	3	5	3	5	3	8	3	9
36	4	9	5	0	5	2	5	4
42	6	3	6	6	6	8	7	0
48	7	7	8	0	8	4	8	6
54	9	2	9	5	9	9	10	2
60	10	5	10	9	11	3	11	7
66	11	7	12	2	12	6	13	0
72	12	7	13	2	13	6	14	1
78	13	4	13	9	14	4	14	9
84	13	9	14	4	14	9	15	4
90	14	1	14	6	15	1	15	6

TABLE V.

Of the Depreſſion of the Horizon

Heighth of the Eye	Depreſſion of the Horizon	
Feet	′	″
1	0	57
2	1	21
3	1	39
4	1	55
5	2	8
6	2	20
7	2	31
8	2	42
9	2	52
10	3	1
12	3	18
14	3	34
16	3	49
18	4	3
20	4	16
22	4	28
24	4	40
26	4	52
28	5	3
30	5	14
35	5	39
40	6	2
45	6	24
50	6	44
55	7	5
60	7	23
65	7	42
70	7	59
75	8	16
80	8	32
85	8	48
90	9	3
95	9	18
100	9	36

PROBLEM II.

The Moon's Horizontal Parallax being 58, to find her Parallax in Altitude.

From the Logarithmic Co-ſine of the Altitude corrected for Refraction, ſubtract the Logarithmic Co-Tang. of 58, the Remainder is the Logar. Sine of the Parallax in Altitude.

EXAMPLE.

Required the Parallax in Altitude at 20
Cos 20 0— 9,97299
Cot. 0 58—11,77280
8,20019=S. 54 31

TABLE

TABLE VI.

For finding the Parallax in Altitude, the horizontal Parallax being 58. Argument, Altitude corrected for Refraction.

Altitude corrected for Refra.	L.	Altitude corrected for Refra.	L.	Altitude corrected for Refra.	L.
1	8 2271	31	8 1603	61	7 9128
2	8 2269	32	8 1556	62	7 8988
3	8 2266	33	8 1508	63	7 8842
4	8 2261	34	8 1458	64	7 8690
5	8 2255	35	8 1406	65	7 8531
6	8 2248	36	8 1352	66	7 8365
7	8 2239	37	8 1295	67	7 8191
8	8 2229	38	8 1237	68	7 8008
9	8 2218	39	8 1177	69	7 7815
10	8 2205	40	8 1114	70	7 7612
11	8 2191	41	8 1050	71	7 7398
12	8 2176	42	8 0983	72	7 7172
13	8 2159	43	8 0913	73	7 6931
14	8 2141	44	8 0841	74	7 6675
15	8 2121	45	8 0767	75	7 6402
16	8 2100	46	8 0690	76	7 6109
17	8 2078	47	8 0610	77	7 5793
18	8 2054	48	8 0527	78	7 5451
19	8 2029	49	8 0441	79	7 5078
20	8 2002	50	8 0353	80	7 4669
21	8 1973	51	8 0261	81	7 4215
22	8 1944	52	8 0165	82	7 3708
23	8 1912	53	8 0067	83	7 3130
24	8 1879	44	7 9964	84	7 2464
25	8 1845	55	7 9858	85	7 1675
26	8 1809	56	7 9748	86	7 0708
27	8 1771	57	7 9633	87	7 9460
28	8 1731	58	7 9514	88	6 7700
29	8 1690	59	7 9390	89	6 4691
30	8 1647	60	7 9262	90	6

TABLE VII.

Of Logarithmic Differences for readily computing the Moon's Parallax in Altitude.

Horizontal Parallax	L. Dif.	Horizontal Parallax	L. Dif.	Horizontal Parallax	L. Dif.
53 0	—0392	56 0	—0152	59 0	+0074
5	0385	5	0146	5	0080
10	0378	10	0140	10	0086
15	0371	15	0134	15	0092
20	0365	20	0127	20	0099
25	0358	25	0121	25	0105
30	0351	30	0114	30	0111
35	0344	35	0108	35	0117
40	0337	40	0101	40	0123
45	0331	45	0094	45	0129
50	0324	50	0088	50	0135
55	0317	55	0082	55	0141
54 0	0310	57 0	0076	60 0	0147
5	0304	5	0070	5	0153
10	0297	10	0063	10	0159
15	0291	15	0057	15	0165
20	0284	20	0050	20	0171
25	0277	25	0043	25	0177
30	0270	30	0038	30	0183
35	0264	35	0032	35	0189
40	0257	40	0025	40	0195
45	0251	45	0019	45	0201
50	0244	50	0013	50	0207
55	0238	55	0007	55	0213
55 0	0231	58 0	+0000	61 0	0219
5	0225	5	0006	5	0225
10	0218	10	0012	10	0231
15	0211	15	0018	15	0237
20	0204	20	0025	20	0243
25	0198	25	0031	25	0249
30	0191	30	0037	30	0254
35	0185	35	0043	35	0260
40	0178	40	0050	40	0266
45	0172	45	0056	45	0272
50	0165	50	0062	50	0278
55	0159	55	0068	55	0284

PROBLEM III.

From the Moon's Apparent, or obſerved Altitude, to find her true Altitude.

Suppoſe the obſerved Altitude was	8	20	0
The Parallax by the Ephemeris	0	54	20
The Latitude of the Place about	43	0	0
The Height above the Surface of the Sea	50 Feet		
The Moon's Semidiameter	0	15	39
The Barometer ſtanding at	31	4 Inch	
And the Thermometer		45	
The lower Limb being in Contact with the Horizon.			
From the apparent Altitude	8	20	0
Subtract the Depreſſion from Table V.		6	44
	8	13	16
Add the Moon's Semidiam	0	15	39
	8	28	55
Subtract the Refraction Table III.		6	9
	8	22	46
But the Barometer is 1.8 Inch higher than the mean State, and the Ther. 5 lower; wherefore I ſay, 9-10ths Inch, : 11 the Variation for the Barometer taken from the third Column : : 1 8 : 22, which muſt be added, becauſe the Barometer is higher			22
	8	23	8

Then

Then from the 4th Column againſt 8 28 55, I take out 17 5, the Variation for 20 of the Thermometer, and ſay 20 : 17 5 : : 5 : 4.4	
Which muſt alſo be added, becauſe the Thermometer is lower than the mean State but ſhould be ſubtracted if it was higher —— ——	0 0 4.4
Gives the Altitude corrected for Refraction —— ——	8 23 12.4
With the equatorial Parallax 54 20 on the Top, and the Latitude 43 in the left Hand Column of Table IV. take out the Correction 6. 6, which muſt always be ſubtracted from the given Parallax.	
In this Caſe it will leave ——	0 54 13.4
With the Altitude above corrected enter Table VI. and take out the Logarithm from Column L.	8,2225
With the reduced Parallax 54 13.4 enter Table VII. and take out the Log. L with its Sign —— ——	— ,0294
	S 8,1931
The Aggregate is the Log. Sine of the true Parallax in Altitude	53 38
Which muſt always be added to the Altitude before corrected	8,23 12.4
The Sum is the true Alt. of the Moon	9,16 50.4

I have been very particular in including all the Elements that can affect the Moon's Altitude, but perhaps they will ſeldom be all uſed in Practice. On Land, where the Altitude is beſt taken by the Reflection of the Object, in ſome undiſturbed Liquid with the Antimeter; or, where perhaps the Obſervation is made with a well fixt Mural Quadrant, the Correction depending on the Depreſſion of the Horizon will be omitted; and at Sea the Barometer and Thermometer will frequently be diſregarded.

The Table of the Depreſſion of the Horizon is here included, rather that the Elements might be compleat, than for its real Utility; for the Refraction of the Horizon itſelf is ſo various in different Latitudes, that it will be erroneous; it is therefore to be recommended frequently to find the Quantity of that Correction by Obſervation, after the following Method.

Let two Obſervers, with their Inſtruments well adjuſted, place themſelves back to back, and both meaſure the Arch between the ſame Object, and the reſpective Horizon of the Sea at the ſame Time.——

Suppoſe the Object was the Sun or Moon, and each Obſerver brought the neareſt Limb in Contact with the Horizon. Add the two Obſervations together, and to them the Sun or Moon's whole Diameter for the Time; from half the Sum ſubtract 90, the Remainder is the Dip of the Horizon.

EXAMPLE.

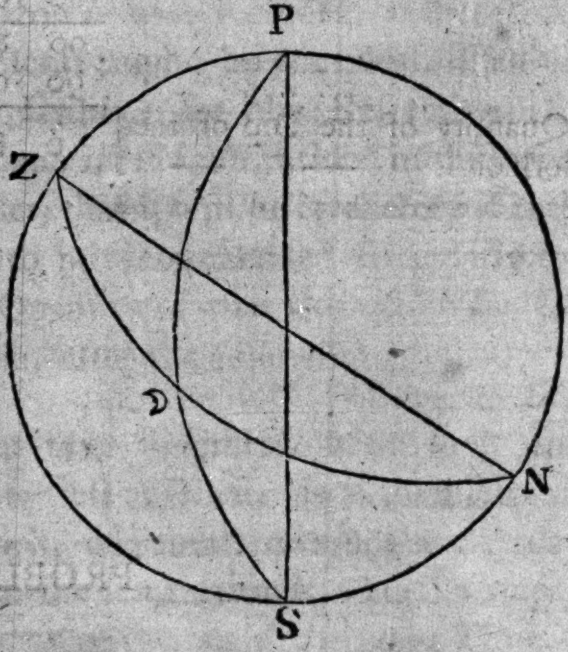
Suppoſe the one Obſervation of the Moon's Diſtance from the Horizon was — —	106	10	0
And the other — —	73	34	24
The Moon's Diameter being —	0	31	40
	180	16	4
	90	8	2
	90	0	0
The Quantity of the Dip of the Horizon is — —	0	8	2

PROBLEM

PROBLEM IV.

From the Latitude of the Place, the Longitude by Account, the Moon's Altitude, and the Time by the Meredian of the Place of Obſervation, to determine the Longitude.

Let P repreſent the neareſt Pole, according as the Latitude is North or South, and Z the Zenith of the Place of Obſervation. Then P☽S will be a Meredian Circle, and PZS the Meredian itſelf. Z☽ will be the Co-Altitude, PZ the Co-Latitude, and P☽ the Moon's Co-Declination.



Here are the three Sides PZ, P☽ and Z☽ given, to find the Angle ZP☽ which turned into Time, including the Moon's Motion in her Orbit, will give the Time of her paſſing the Meredian of the Place of Obſervation; which Time, compared with the Time of her paſſing the Meredian of *Greenwich*, allowing for her Motion in her Orbit, will determine the Difference of the Meredians of *Greenwich*, and the Place of Obſervation.

EXAMPLE.

March 7th, 1781, at $11^{h}\ 3^{m}$ apparent Time, in Latitude 51 29 40, Longitude ſuppoſed about 30 Seconds of Time Weſt, the Altitude of the Moon's lower Limb was obſerved 56° 30′ 30.″ Required the Longitude.

From the Time at the Place of Obſervation, and ſuppoſed Longitude, I take the *Greenwich* Time to be —— —— ——	11	3	30
Obſerved Altitude	56	30	30
☽'s Semidiameter	0	16	28.6
	56	46	58.6
Refraction			38
	56	46	20.6

Hori-

Horizontal Parallax by the Ephemeris			60 27
With which, and the given Latitude, I take out its Reduction from Table IV. — — —			9. 5
			60 17. 5
With the Altitude and the Horizontal Parallax thus corrected, take the Log from Tables VI and VII.			7,9668 +0168
And the Parallax in Altitude is 33 6			S=7,9836
Which added to the Altitude before corrected, gives the true Altit.			57 19 27
		Co-Alt.	32 40 33
S. Co-Ar.	0,2057975	Co-Lat.	38 30 20
S. Co-Ar.	0,0301570	Co-Declin.	68 53 39
			140 4 32
S. Log.	9,9730899	— —	70 2 16
S. Log.	9,7830800	— —	37 21 43
	19,9921244		
	9,9960622=S.	82 17 46	
		164 35 32	
		15 24 28	Horary Angle
Now becauſe the Moon is ſo much paſt the Meredian, I take her hourly Motion from the Ephemeris in right Aſcenſion. — — —			0 37 35
And ſubtract it from —			15 0 0
The Remainder is her hourly Motion from the Meredian			14 22 25
m			h m s
Then 14 22 25 : 60 : : 15 24 28 :			1 4 10
Which taken from the Time of Obſervation — —			11 3 0
Leaves the Time of the Moon paſſing the Meredian			9 58 50

But by Calculation it appears the Moon did paſs the Meredian of *Greenwich* at — — } 9 59 22

The Difference is — — 0 0 32

Then becauſe this Time is to the Eaſt, or earlier than *Greenwich* Time, I take the Moon's hourly Motion in right Aſcenſion, and ſubtract it from 15, the Difference is — } 14 22 25

And ſay $15^{\circ} : 14\ 22\ 25 :: 32^{s} : 30^{s}\ 6$]

Which gives the Longitude Eaſt 7′ 39″

But the true Longitude of the Place of Obſervation, by many Experiments & Proofs, hath been ſettled at — } 6 45 Weſt

The Sum is — — 14 24

Which Error in this Caſe is under 10 Miles.

A nearer Determination cannot be expected, unleſs extraordinary Attention was paid in computing the Time of Obſervation: But as the preceding Example was one of ſix progreſſive Obſervations, which all agreed in the Concluſion within ſix Seconds of Time, it evinces that the Error was in the Time Piece, and that *this Method of finding the Longitude at Sea*, may be depended on frequently to that Diſtance, and generally to twelve Seconds of Time, without the Trouble of a double and critical Calculus. What now remains to render it compleat, is the Time of the Moon's paſſing the Meredian of *Greenwich* to be carried to Seconds in the Ephemeris.

FINIS.